8 The Minimal Supersymmetric Standard Model: Part 2

8.1 Electroweak Symmetry Breaking

$$\begin{aligned} V(H_u, H_d) &= (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\ &\quad + b\,(H_u^+ H_d^- - H_u^0 H_d^0) + h.c. \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\ &\quad + \frac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2. \end{aligned} \tag{8.1}$$

$$\begin{aligned} V(H_u^0, H_d^0) &= (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b\,H_u^0 H_d^0 + h.c.) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2. \end{aligned} \tag{8.2}$$

To destabilize the origin we need:

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2). \tag{8.3}$$

To ensure the potential is bounded from below we need:

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2. \tag{8.4}$$

These relations show that there is a tight relation between the soft SUSY breaking parameters and the SUSY preserving μ-term. A prioi these parameters should be unrelated. This is known as the "μ problem". Solutions to this problem require μ to vanish at tree level and be produced as a by-product of SUSY breaking [5, 6, 7] for examples. .

$$\langle H_u^0 \rangle = \frac{v_u}{\sqrt{2}} \tag{8.5}$$

$$\langle H_d^0 \rangle = \frac{v_d}{\sqrt{2}} \tag{8.6}$$

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2 \tag{8.7}$$

$$v^2 = v_u^2 + v_d^2 \approx (246\,\text{GeV})^2 \tag{8.8}$$

Figure 1: The μ-problem for $m_{H_u}^2 = -\frac{1}{2} m_{H_d}^2$.

$$\sin\beta \equiv \frac{v_u}{v} \tag{8.9}$$
$$\cos\beta \equiv \frac{v_d}{v} \tag{8.10}$$
$$\tan\beta = v_u/v_d \tag{8.11}$$
$$\cos 2\beta = \frac{v_d^2 - v_u^2}{v^2} \tag{8.12}$$
$$0 < \beta < \pi/2 \tag{8.13}$$

The minimum conditions $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ give

$$|\mu|^2 + m_{H_d}^2 = b\tan\beta - (m_Z^2/2)\cos 2\beta; \tag{8.14}$$
$$|\mu|^2 + m_{H_u}^2 = b\cot\beta + (m_Z^2/2)\cos 2\beta. \tag{8.15}$$

this is another way of seeing the "μ problem"

The Higgs scalar fields consist of eight real scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be Nambu-Goldstone bosons π^0, $\pi^\pm$ which are eaten by the Z^0 and $W^\pm$. This leaves five degrees of freedom A^0,$H^\pm$, h^0, and H^0. Shift fields by vevs:

$$H_u^0 \to \frac{v_u}{\sqrt{2}} + H_u^0 \tag{8.16}$$
$$H_d^0 \to \frac{v_d}{\sqrt{2}} + H_d^0 \tag{8.17}$$

$$V \supset (\mathrm{Im}H_u^0, \mathrm{Im}H_d^0) \left(\begin{array}{cc} b\cot\beta & b \\ b & b\tan\beta \end{array} \right) \left(\begin{array}{c} \mathrm{Im}H_u^0 \\ \mathrm{Im}H_d^0 \end{array} \right) \tag{8.18}$$

$$\begin{pmatrix} \pi^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im}H_u^0 \\ \mathrm{Im}H_d^0 \end{pmatrix}, \tag{8.19}$$

$$m_A^2 = \frac{b}{s_\beta c_\beta} \tag{8.20}$$

$$V \supset (H_u^{+*}, H_d^-) \begin{pmatrix} b\cot\beta + m_W^2 c_\beta^2 & b + m_W^2 c_\beta s_\beta \\ b + m_W^2 c_\beta s_\beta & b\tan\beta + m_W^2 s_\beta^2 \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} \tag{8.21}$$

$$\begin{pmatrix} \pi^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix}, \tag{8.22}$$

$\pi^- = \pi^{+*}$ and $H^- = H^{+*}$

$$m_{H^\pm}^2 = \quad m_A^2 + m_W^2 \tag{8.23}$$

$$V \supset (\mathrm{Re}H_u^0, \mathrm{Re}H_d^0) \begin{pmatrix} b\cot\beta + m_Z^2 s_\beta^2 & -(b + m_Z^2) c_\beta s_\beta \\ -(b + m_Z^2) c_\beta s_\beta & b\tan\beta + m_Z^2 c_\beta^2 \end{pmatrix} \begin{pmatrix} \mathrm{Re}H_u^0 \\ \mathrm{Re}H_d^0 \end{pmatrix} \tag{8.24}$$

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{Re}H_u^0 \\ \mathrm{Re}H_d^0 \end{pmatrix}. \tag{8.25}$$

$$m_{h,H}^2 = \frac{1}{2}\left(m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4 m_Z^2 m_A^2 \cos^2 2\beta} \right). \tag{8.26}$$

the mixing angle α is determined given by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_A^2 + m_Z^2}{m_H^2 - m_h^2}; \qquad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_A^2 - m_Z^2}{m_H^2 - m_h^2}. \tag{8.27}$$

m_A, $m_H^\pm$, and $m_H \to \infty$ as $b \to \infty$ but m_h is maximized at $m_A = \infty$

$$m_h < |\cos 2\beta| m_Z \tag{8.28}$$

there are however large one-loop corrections that we will see later. For $m_{A^0} \gg m_Z$, A^0, H^0, and $H^\pm$ are much heavier than h^0, forming a nearly degenerate isospin doublet. In this limit, the angle α is fixed to be approximately $\beta - \pi/2$, and h^0 has Standard Model couplings to quarks, leptons, and gauge bosons.

8.2 The Sparticle Spectrum: Squarks and Sleptons

In general we have to diagonalize 6×6 matrices since all scalars with the same quantum numbers mix. Neglecting mixing for the third generation, we have for the top squarks

$$-\mathcal{L} \supset (\,\widetilde{t}_L^* \quad \widetilde{t}_R^*\,)\,\mathbf{m}^2_{\widetilde{\mathbf{t}}} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix} \tag{8.29}$$

where

$$\mathbf{m}^2_{\widetilde{\mathbf{t}}} = \begin{pmatrix} m^2_{Q_3} + m_t^2 + D_u & v(a_t s_\beta - \mu y_t c_\beta) \\ v(a_t s_\beta - \mu y_t c_\beta) & m^2_{\overline{u}_3} + m_t^2 + D_{\overline{u}} \end{pmatrix}, \tag{8.30}$$

and

$$D_\phi = (T_3^\phi - Q^\phi_{\rm EM} \sin^2\theta_W) \cos 2\beta\, m_Z^2. \tag{8.31}$$

diagonalize to give mass eigenstates $\widetilde{t}_1$ and $\widetilde{t}_2$ with $m^2_{\widetilde{t}_1} < m^2_{\widetilde{t}_2}$. Mixing angles then appear in vertices for mass eigenstates.

Similarly for bottom squarks and tau sleptons (in their gauge-eigenstate bases $(\widetilde{b}_L, \widetilde{b}_R)$ and $(\widetilde{\tau}_L, \widetilde{\tau}_R)$)

$$\mathbf{m}^2_{\widetilde{\mathbf{b}}} = \begin{pmatrix} m^2_{Q_3} m_b^2 + D_d & v(a_b c_\beta - \mu y_b s_\beta) \\ v(a_b c_\beta - \mu y_b s_\beta) & m^2_{\overline{d}_3} + m_b^2 + D_{\overline{d}} \end{pmatrix}; \tag{8.32}$$

$$\mathbf{m}^2_{\widetilde{\tau}} = \begin{pmatrix} m^2_{L_3} + m_\tau^2 D_e & v(a_\tau c_\beta - \mu y_\tau s_\beta) \\ v(a_\tau c_\beta - \mu y_\tau s_\beta) & m^2_{\overline{e}_3} + m_\tau^2 + D_{\overline{e}} \end{pmatrix} \tag{8.33}$$

Note that large masses for third generation particles allow for large mixing in these matrices and the possibility that the lower eigenvalue is driven negative. This would give vevs to squarks and sleptons which can break $U(1)_{\rm em}$ and $SU(3)_c$.

It is interesting to note what would have happened if we didn't have soft SUSY breaking mass terms

$$\mathbf{m}^2_{\widetilde{\mathbf{u}}} = \begin{pmatrix} \mathbf{m_u}^\dagger \mathbf{m_u} + D_u \mathbf{I} & \boldsymbol{\Delta}_u \\ \boldsymbol{\Delta}_\mathbf{u}^\dagger & \mathbf{m_u} \mathbf{m_u}^\dagger + D_{\overline{u}} \mathbf{I} \end{pmatrix} \tag{8.34}$$

$$\mathbf{m}^2_{\widetilde{\mathbf{d}}} = \begin{pmatrix} \mathbf{m_d}^\dagger \mathbf{m_d} + D_d \mathbf{I} & \boldsymbol{\Delta}_\mathbf{d} \\ \boldsymbol{\Delta}_\mathbf{d}^\dagger & \mathbf{m_d} \mathbf{m_d}^\dagger + D_{\overline{d}} \mathbf{I} \end{pmatrix} \tag{8.35}$$

Note that $D_u + D_{\overline{u}} + D_d + D_{\overline{d}} = 0$, so at least one $D_\phi \leq 0$. Suppose $D_u \leq 0$, let

$$m_u \gamma = m_0 \gamma \tag{8.36}$$

where m_0 is the smallest eigenvalue of m_u then

$$(\gamma^T, 0)\mathbf{m}^{\mathbf{2}}_{\tilde{\mathbf{u}}} \begin{pmatrix} \gamma \\ 0 \end{pmatrix} \leq m_0^2 \tag{8.37}$$

So there would be a squark lighter than the u or d quarks [8]

8.3 The Sparticle Spectrum: Charginos

$\widetilde{W}^\pm$ and $\widetilde{H}_u^\pm$ mix, their mass eigenstates are called *charginos*. In basis $\psi^\pm = (\widetilde{W}^+, \widetilde{H}_u^+, \widetilde{W}^-, \widetilde{H}_d^-)$, the chargino mass terms are

$$\mathcal{L} \supset -\frac{1}{2}(\psi^\pm)^T \mathbf{M}_{\widetilde{C}} \psi^\pm + h.c. \tag{8.38}$$

where

$$\mathbf{M}_{\widetilde{C}} = \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix}; \qquad \mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} s_\beta \, m_W \\ \sqrt{2} c_\beta \, m_W & \mu \end{pmatrix}. \tag{8.39}$$

diagonalize by

$$\mathbf{U}^* \mathbf{X} \mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix}. \tag{8.40}$$

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}; \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}. \tag{8.41}$$

U and **V** appear in the interaction vertices for chargino mass eigenstates.

$$\begin{aligned} m^2_{\widetilde{C}_1, \widetilde{C}_2} &= \frac{1}{2}\Big[(|M_2|^2 + |\mu|^2 + 2m_W^2) \\ &\mp \sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2}\Big] \end{aligned} \tag{8.42}$$

In the limit that $||\mu| \pm M_2| \gg m_W$ the charginos are approxiamtely a wino and a higgsino with masses $|M_2|$ and $|\mu|$.

8.4 The Sparticle Spectrum: Neutralinos

$\psi^0 = (\widetilde{B}, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0)$, all mix with each other form four neutral mass eigenstates called *neutralinos*: $\widetilde{N}_i$ $(i = 1, 2, 3, 4)$ and $m_{\widetilde{N}_1} < m_{\widetilde{N}_2} < m_{\widetilde{N}_3} < m_{\widetilde{N}_4}$

the neutralino mass terms in the lagrangian are

$$\mathcal{L} \supset -\frac{1}{2}(\psi^0)^T \mathbf{M}_{\widetilde{N}} \psi^0 + h.c. \tag{8.43}$$

where

$$\mathbf{M}_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -c_\beta\, s_W\, m_Z & s_\beta\, s_W\, m_Z \\ 0 & M_2 & c_\beta\, c_W\, m_Z & -s_\beta\, c_W\, m_Z \\ -c_\beta\, s_W\, m_Z & c_\beta\, c_W\, m_Z & 0 & -\mu \\ s_\beta\, s_W\, m_Z & -s_\beta\, c_W\, m_Z & -\mu & 0 \end{pmatrix}. \tag{8.44}$$

diagonalized by a unitary matrix $\mathbf{N}$

$$\mathbf{M}_{\widetilde{N}}^{\text{diag}} = \mathbf{N}^* \mathbf{M}_{\widetilde{N}} \mathbf{N}^{-1} \tag{8.45}$$

for

$$m_Z \ll |\mu \pm M_1|, |\mu \pm M_2| \tag{8.46}$$

then the neutralino mass eigenstates are very nearly $\widetilde{B}$, $\widetilde{W}^0$, $(\widetilde{H}_u^0 \pm \widetilde{H}_d^0)/\sqrt{2}$, with mass eigenvalues: $(M_1, N_2, |\mu|, |\mu|)$. A "bino-like" LSP can make a good dark matter candidate, and N_1 is often assumed to be the LSP.

8.5 The Sparticle Spectrum: Gluinos

The gluino is a color octet fermion so it can't mix with anything, it's mass is just given by the soft SUSY breaking mass M_3.

References

[1] S.P. Martin, "A supersymmetry primer," hep-ph/9709356.

[2] M. Drees, "An introduction to supersymmetry," hep-ph/9611409.

[3] C. Csaki, "The minimal supersymmetric standard model (MSSM)," Mod. Phys. Lett. **A11** (1996) 599 hep-ph/9606414.

[4] H.E. Haber, "Introductory low-energy supersymmetry," hep-ph/9306207.

[5] J.E. Kim and H. P. Nilles, *Phys. Lett.* B **138**, 150 (1984) J.E. Kim and H. P. Nilles, *Phys. Lett.* B **263**, 79 (1991); E.J. Chun, J.E. Kim and H.P. Nilles, *Nucl. Phys.* **B370**, 105 (1992).

[6] G.F. Giudice and A. Masiero, *Phys. Lett.* B **206**, 480 (1988); J.A. Casas and C. Muñoz, *Phys. Lett.* B **306**, 288 (1993).

[7] G. Dvali, G.F. Giudice and A. Pomarol, *Nucl. Phys.* **B478**, 31 (1996).

[8] S. Dimopoulos and H. Georgi, "Softly Broken Supersymmetry And SU(5)," Nucl. Phys. **B193** (1981) 150.